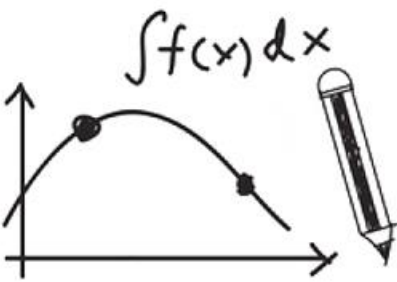


Calculus(I)

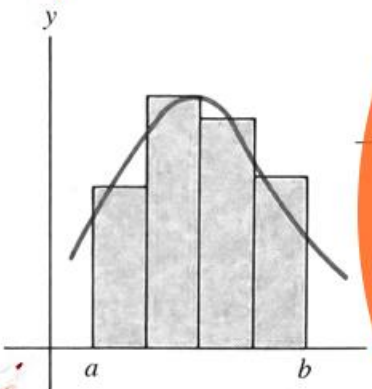
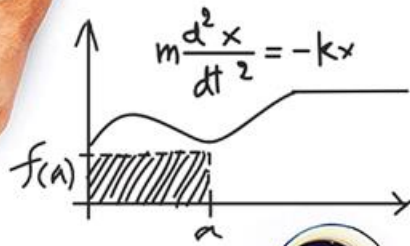
$$x^2 - 3x - 4 = 0$$

$$4x^2 - 3x - 1 = 0$$



$$\frac{d}{dx} \left[\frac{f(x)}{g(x)} \right] = \frac{g(x)f'(x) - f(x)g'(x)}{g(x)^2}$$

$$F = mg = ma = m \frac{d^2h}{dt^2}$$



Gottfried Wilhelm Leibniz

$$\frac{dA}{dt} = \frac{dB}{dt} = -\frac{dC}{dt} = \frac{dD}{dt} = (c_1)T^{\frac{1}{2}}AB - (c_2)T^{\frac{1}{2}}CD$$

$$m \frac{d^2x}{dt^2} = -kx - f \frac{dx}{dt} + A \sin(\omega t)$$

$$y' = \text{and } v' = -ky - fv + A \sin(\omega t)$$

$$m \frac{d^2x}{dt^2} = -kx$$

$$x = A \frac{dT}{dt} - (c_1)(T - T)$$



$$\frac{b^2 - 4ac}{4a^2} \quad x + \frac{b}{2a} = \frac{\sqrt{b^2 - 4ac}}{2a} \quad x + \frac{b}{2a} = -\frac{\sqrt{b^2 - 4ac}}{2a}$$



$$x + h, f(x + \tau)$$



Basic Integration Rules

Lecturer: Xue Deng

How to compute this kind of the integral ?

$$\int \sin x dx = -\cos x + C$$

known

$$\int \sin 2x dx = -\cos 2x + C$$



$$\int \sin 2x dx = ?$$

Substitution in Indefinite Integrals

Let g be a differentiable function and suppose that F is an anti-derivative of f . Then, if $u=g(x)$,

$$\begin{aligned}\int f(g(x))g'(x)dx &= \int \underline{f(g(x))} \underline{dg(x)} \\ &= \int f(u)du \\ &= F(u) + C = F(g(x)) + C.\end{aligned}$$

Substitution in Indefinite Integrals

$$\int f[\varphi(x)] \varphi'(x) dx = \int f[\varphi(x)] d\varphi(x)$$

$$\underline{\underline{u = \varphi(x)}} \left[\int f(u) du \right]_{u=\varphi(x)}$$



substitution in
indefinite integrals

The general principle :


(1) $u = \varphi(x) \Rightarrow \int f(u) du$

(2) replace $\varphi(x)$ for u in $F(u) + C \Rightarrow F(\varphi(x)) + C.$

Example 1

$$\int \sin x dx = -\cos x + C$$

Find $\int \sin 2x dx$

 **Method 1** $= \frac{1}{2} \int \sin 2x d(2x) \underline{\underline{u = 2x}}$

$$= \frac{1}{2} \int \sin u du$$
$$= -\frac{1}{2} \cos u + C$$
$$= -\frac{1}{2} \cos 2x + C$$

Example 1

$$\int x^u dx = \frac{x^{u+1}}{u+1} + C$$

Find $\int \sin 2x dx$



Method 2 $= 2 \int \sin x \cos x dx$


$= 2 \int \sin x d(\sin x) \quad \underline{\underline{u = \sin x}} = 2 \int u du$

$= u^2 + C$

$= (\sin x)^2 + C$

Example 1

Find $\int \sin 2x dx$

 **Method 3** = $2 \int \sin x \cos x dx$

$= -(\cos x)^2 + C$

Note: A pink box highlights 'sin x' in the first equation, and a pink arrow points from it to the 'cos x' term.

Example 1

Find $\int \sin 2x dx$



Method1 $= -\frac{1}{2} \cos 2x + C$


Method2 $= (\sin x)^2 + C$

Method3 $= -(\cos x)^2 + C$

Example 2

$$\int \frac{1}{x} dx = \ln|x| + C$$

Find $\int \frac{1}{3+2x} dx$

 $= \frac{1}{2} \int \frac{1}{3+2x} d(3+2x)$ $u = 3+2x$

$$= \frac{1}{2} \int \frac{1}{u} du = \frac{1}{2} \ln|u| + C$$


$$= \frac{1}{2} \ln|3+2x| + C$$

Example 3

$$\int \frac{1}{1+x^2} dx = \arctan x + C$$

Find $\int \frac{1}{a^2 + x^2} dx$


$$\left(u = \frac{x}{a} \right)$$


$$\begin{aligned} &= \frac{1}{a^2} \int \frac{1}{1 + \frac{x^2}{a^2}} dx = \frac{1}{a^2} \int \frac{1}{1 + \left(\frac{x}{a}\right)^2} dx \\ &= \frac{1}{a} \int \frac{1}{1 + \left(\frac{x}{a}\right)^2} d\left(\frac{x}{a}\right) = \frac{1}{a} \int \frac{1}{1 + u^2} d(u) \\ &= \frac{1}{a} \arctan u + C = \frac{1}{a} \arctan \frac{x}{a} + C \end{aligned}$$

Example 4

By Eg.3 $\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \arctan \frac{x}{a} + C$

Find $\int \frac{1}{x^2 - 8x + 25} dx$

 $= \int \frac{1}{(x-4)^2 + 9} dx$

$$= \int \frac{1}{3^2 + (x-4)^2} dx = \int \frac{d(x-4)}{3^2 + (x-4)^2}$$

$$= \frac{1}{3} \arctan \frac{x-4}{3} + C$$

Summary

Substitution in indefinite integrals

$$\int f(ax + b)dx = \frac{1}{a} \int f(ax + b)d(ax + b) (a \neq 0)$$

$$\int f(ax^{m+1} + b)x^m dx = \frac{1}{a(m+1)} \int f(ax^{m+1} + b)d(ax^{m+1} + b)$$

$$\int f\left(\frac{1}{x}\right) \frac{dx}{x^2} = - \int f\left(\frac{1}{x}\right) d\left(\frac{1}{x}\right) \quad \int f(\ln x) \frac{1}{x} dx = \int f(\ln x) d(\ln x)$$

$$\int f(e^x) e^x dx = \int f(e^x) d(e^x) \quad \int f(\sqrt{x}) \frac{dx}{\sqrt{x}} = 2 \int f(\sqrt{x}) d(\sqrt{x})$$

Summary

Substitution in indefinite integrals

$$\int f(\sin x) \cos x dx = \int f(\sin x) d \sin x \quad \int f(\cos x) \sin x dx = -\int f(\cos x) d \cos x$$

$$\int f(\tan x) \sec^2 x dx = \int f(\tan x) d \tan x \quad \int f(\cot x) \csc^2 x dx = -\int f(\cot x) d \cot x$$

$$\int f(\arcsin x) \frac{1}{\sqrt{1-x^2}} dx = \int f(\arcsin x) d \arcsin x$$


$$\int f(\arctan x) \frac{1}{\sqrt{1+x^2}} dx = \int f(\arctan x) d \arctan x$$

$$\int \frac{f'(x)}{f(x)} dx = \int \frac{df(x)}{f(x)} = \ln |f(x)| + C$$

Questions and Answers

$$\int f(\ln x) \frac{1}{x} dx = \int f(\ln x) d(\ln x)$$

? $\int \frac{\ln x}{x} dx$

 $= \int \ln x d(\ln x)$

$$= \frac{(\ln x)^2}{2} + C$$

Questions and Answers

$$? \int \frac{1}{x(1+2\ln x)} dx$$

$$\text{✎} = \int \frac{1}{1+2\ln x} d(\ln x)$$


$$= \frac{1}{2} \int \frac{1}{1+2\ln x} d(1+2\ln x)$$

$$= \frac{1}{2} \ln |1+2\ln x| + C$$

Questions and Answers

$$\int \frac{1}{\sqrt{1-x^2}} dx = \arcsin x + C$$

? $\int \frac{1}{\sqrt{a^2 - x^2}} dx (a > 0)$


$$= \int \frac{dx}{\sqrt{a^2 \left(1 - \frac{x^2}{a^2}\right)}} = \frac{1}{a} \int \frac{a d\left(\frac{x}{a}\right)}{\sqrt{1 - \left(\frac{x}{a}\right)^2}}$$
$$= \int \frac{\frac{a}{a} d\left(\frac{x}{a}\right)}{\sqrt{1 - \left(\frac{x}{a}\right)^2}} = \arcsin \frac{x}{a} + C$$

Basic Integration Rules

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