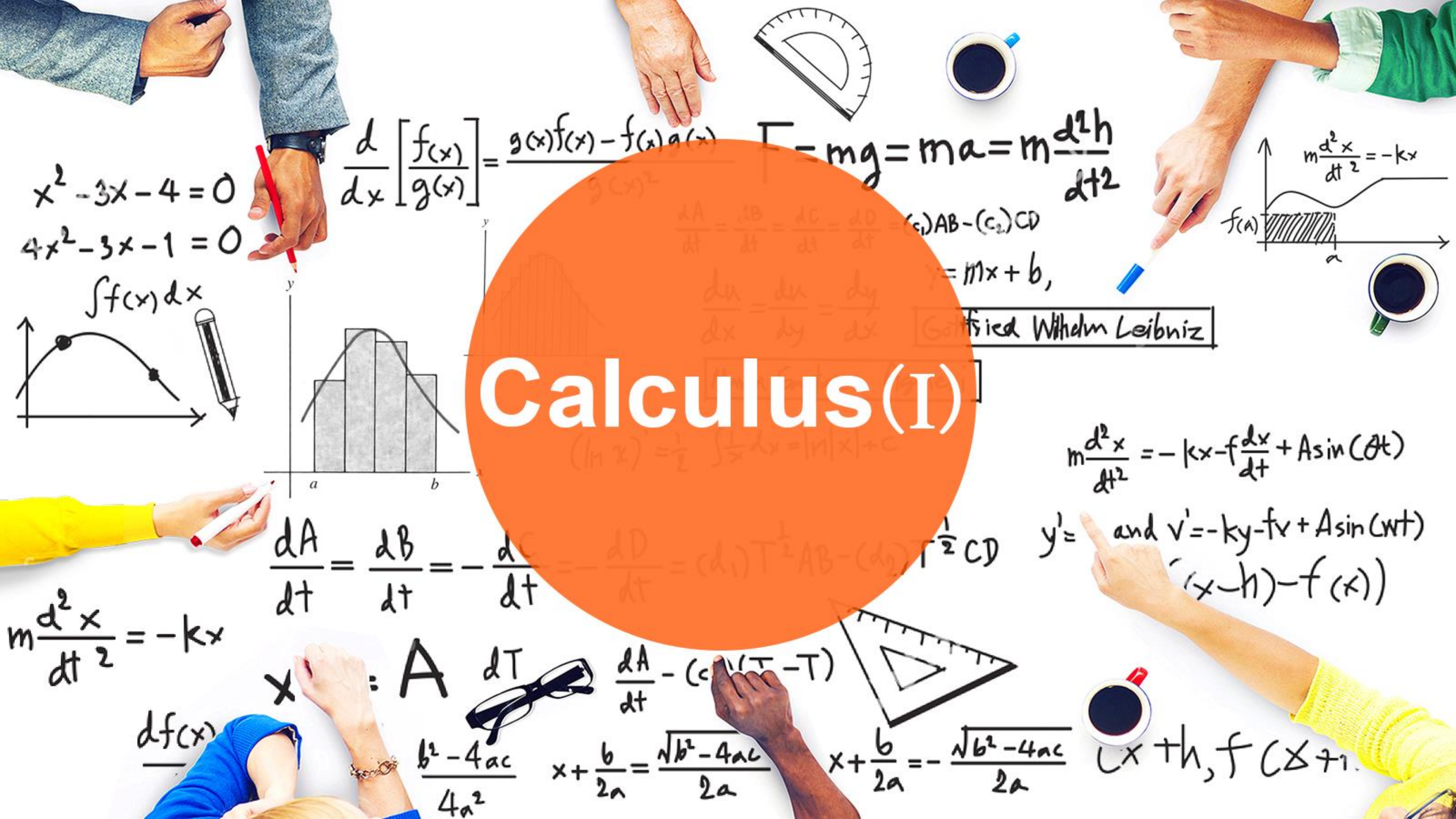

Calculus(Ⅰ)

Basic Integration Rules

Lecturer: Xue Deng

How to compute this kind of the integral ?

$$\int \sin x dx = -\cos x + C$$

known

$$\int \sin 2x dx = -\cos 2x + C$$

$$\int \sin 2x dx =$$

Substitution in Indefinite Integrals

Let g be a differentiable function and suppose that F is an anti-derivative of f. Then, if $u=g(x)$,

$$\int f(g(x))g'(x)dx = \int f(\underline{g(x)})d\underline{g(x)}$$

$$= \int f(u)du$$

$$= F(u) + C = F(g(x)) + C.$$

Substitution in Indefinite Integrals

$$\int f[\varphi(x)]\varphi'(x)dx = \int f[\varphi(x)]d\varphi(x)$$

$$\overset{u=\varphi(x)}{=\!=} \left[\int f(u)du\right]_{u=\varphi(x)}$$

substitution in indefinite integrals

The general principle :

(1) $u = \varphi(x) \Rightarrow \int f(u)du$

(2) replace $\varphi(x)$ for u in $F(u) + C \Rightarrow F(\varphi(x)) + C.$

Example 1

$\int \sin x dx = -\cos x + C$

Find $\int \sin 2x dx$

Method1 $= \frac{1}{2}\int \sin 2x d(2x)$ $u = 2x$

$= \frac{1}{2}\int \sin u du$

$= -\frac{1}{2}\cos u + C$

$= -\frac{1}{2}\cos 2x + C$

Example 1

$$\int x^u dx = \frac{x^{u+1}}{u+1} + C$$

Find $\int \sin 2x dx$

Method2 $= 2\int \sin x \cos x dx$

$= 2\int \sin x d(\sin x)$ $\underline{\underline{u = \sin x}}$ $= 2\int u du$

$= u^2 + C$

$= (\sin x)^2 + C$

Example 1

Find $\int \sin 2x dx$

Method3 $= 2\int \sin x \cos x dx$

$= -(\cos x)^2 + C$

Example 1

Find $\int \sin 2x dx$

Method1 $= -\frac{1}{2}\cos 2x + C$

Method2 $= (\sin x)^2 + C$

Method3 $= -(\cos x)^2 + C$

Example 2

$$\int \frac{1}{x}dx = \ln|x| + C$$

Find $\int \frac{1}{3+2x}dx$

$$= \frac{1}{2}\int \frac{1}{3+2x}d(3+2x) \quad u = 3+2x$$

$$= \frac{1}{2}\int \frac{1}{u}du = \frac{1}{2}\ln|u| + C$$

$$= \frac{1}{2}\ln|3+2x| + C$$

Example 3

$$\int \frac{1}{1+x^2}dx = \arctan x + C$$

Find $\displaystyle\int \frac{1}{a^2+x^2}dx$ $\left(u = \frac{x}{a}\right)$

$$= \frac{1}{a^2}\int \frac{1}{1+\frac{x^2}{a^2}}dx = \frac{1}{a^2}\int \frac{1}{1+(\frac{x}{a})^2}dx$$

$$= \frac{1}{a}\int \frac{1}{1+(\frac{x}{a})^2}d(\frac{x}{a}) = \frac{1}{a}\int \frac{1}{1+u^2}d(u)$$

$$= \frac{1}{a}\arctan u + C = \frac{1}{a}\arctan \frac{x}{a} + C$$

Example 4

By Eg.3 $\int \frac{1}{a^2+x^2}dx = \frac{1}{a}\arctan\frac{x}{a}+C$

Find $\int \frac{1}{x^2-8x+25}dx$

$$= \int \frac{1}{(x-4)^2+9}dx$$

$$= \int \frac{1}{3^2+(x-4)^2}dx = \int \frac{d(x-4)}{3^2+(x-4)^2}$$

$$= \frac{1}{3}\arctan\frac{x-4}{3}+C$$

Summary

Substitution in indefinite integrals

$$\int f(ax+b)dx = \frac{1}{a}\int f(ax+b)d(ax+b)(a \neq 0)$$

$$\int f(ax^{m+1}+b)x^m dx = \frac{1}{a(m+1)}\int f(ax^{m+1}+b)d(ax^{m+1}+b)$$

$$\int f(\frac{1}{x})\frac{dx}{x^2} = -\int f(\frac{1}{x})d(\frac{1}{x}) \quad \int f(\ln x)\frac{1}{x}dx = \int f(\ln x)d(\ln x)$$

$$\int f(e^x)e^x dx = \int f(e^x)d(e^x) \quad \int f(\sqrt{x})\frac{dx}{\sqrt{x}} = 2\int f(\sqrt{x})d(\sqrt{x})$$

Summary

Substitution in indefinite integrals

$$\int f(\sin x)\cos x dx = \int f(\sin x) d\sin x \qquad \int f(\cos x)\sin x dx = -\int f(\cos x) d\cos x$$

$$\int f(\tan x)\sec^2 x dx = \int f(\tan x) d\tan x \qquad \int f(\cot x)\csc^2 x dx = -\int f(\cot x) d\cot x$$

$$\int f(\arcsin x)\frac{1}{\sqrt{1-x^2}}dx = \int f(\arcsin x) d\arcsin x$$

$$\int f(\arctan x)\frac{1}{\sqrt{1+x^2}}dx = \int f(\arctan x) d\arctan x$$

$$\int \frac{f'(x)}{f(x)}dx = \int \frac{df(x)}{f(x)} = \ln|f(x)| + C$$

Questions and Answers

$$\int f(\ln x)\frac{1}{x}dx = \int f(\ln x)d(\ln x)$$

$$\int \frac{\ln x}{x}dx$$

$$= \int \ln x \, d(\ln x)$$

$$= \frac{(\ln x)^2}{2} + C$$

Questions and Answers

$$\int \frac{1}{x(1+2\ln x)}dx$$

$$= \int \frac{1}{1+2\ln x} d(\ln x)$$

$$= \frac{1}{2} \int \frac{1}{1+2\ln x} d(1+2\ln x)$$

$$= \frac{1}{2}\ln\left|1+2\ln x\right| + C$$

Questions and Answers

$$\int \frac{1}{\sqrt{1-x^2}}dx = \arcsin x + C$$

$$\int \frac{1}{\sqrt{a^2-x^2}}dx (a>0)$$

$$= \int \frac{dx}{\sqrt{a^2(1-\frac{x^2}{a^2})}} = \frac{1}{a}\int \frac{ad(\frac{x}{a})}{\sqrt{1-(\frac{x}{a})^2}}$$

$$= \int \frac{d(\frac{x}{a})}{\sqrt{1-(\frac{x}{a})^2}} = \arcsin\frac{x}{a} + C$$

Basic Integration Rules